

Maze Report: DFS, BFS, A* on Maze

Problem

A random maze is generated with a starting point, goal point, open cells, and obstacles. Three algorithm's (depth first search, breadth first search, and A* search) performance are evaluated in finding a path from the starting point to the goal point through open cells.

Maze Generation

To test the three search algorithms, a maze must be first constructed. At the start of the program, the user provides the number of rows, the number of columns, and the obstacle density of the maze. The obstacle density represents roughly how much percentage of the cells in the maze are obstacles. These variables are used in the `generate_maze` function.

The `generate_maze` function creates four global variables, `start_row`, `start_col`, `goal_row`, and `goal_col`, that assist in creating the maze and the search algorithms. They store the locations of the starting point and goal point. The maze itself is stored in a 2d array of the size specified by the user. Each cell in the 2d array can contain '.', 'X', 'S', or 'G'. The '.' represents an open cell, the 'X' represents an obstacle, the 'S' represents the starting point, and the 'G' represents the goal.

Each cell is appended to the maze one by one until the 2d maze is constructed. Randomness is used to determine what a cell will contain. First, when both the starting point and the goal point have not been set, there is a 4% chance one will be placed. Both have a 50% chance of being chosen to possibly be placed. Next, when one point is remaining to being placed, its probability of being placed depends on the given cell's Manhattan Distance to the set state. At a distance less than 10 cells, the probability of being placed is .5% and 1% elsewhere. This makes it more likely that the starting point and goal point are farther apart. Finally, if neither points are placed, a cell becomes either an open cell or an obstacle. The chance of being an obstacle is the obstacle density given by the user. If either the starting point or goal point are not placed, the maze is iterated through until both are.

Search Algorithms

Depth-First Search

It uses a deque data structure as a stack to determine the next cell to expand and a dictionary to record parent-child relationships. The cell's coordinates are recorded in the stack as a tuple. The algorithm prioritizes visiting a cell's adjacent cells in the order: top, right, bottom, left. A cell can be added to the stack multiple times, and the parent of the cell is always updated to reflect the most recent cell that added it to the stack.

Breadth-First Search

It uses a queue data structure to determine the next cell to expand and a dictionary to record parent-child relationships. The cell's coordinates are recorded in the queue as a tuple. The algorithm prioritizes visiting a cell's adjacent cells in the order: top, right, down, left. A cell can only be added to the queue once. Hence the parent of a cell is the cell that added it to the queue.

A* Search

It uses a priority queue data structure to determine the next cell to expand and a dictionary to record parent-child relationships. The next cell to expand is the cell in the priority queue data structure with the minimum key value. The key value of a cell is its number of steps from the starting point [g(n)] plus its Manhattan Distance [h(n)] to the goal state. In the data section, the cell's coordinates are recorded as a tuple and its step cost from the starting point. The data recorded therefore is in the form: (g(n) + h(n), (row, column), g(n)). The data section is a tuple consisting of a tuple and an integer. Similarly to breadth-first search, a cell can only be added once to the priority queue, and its parent is the cell that added it to the priority queue.

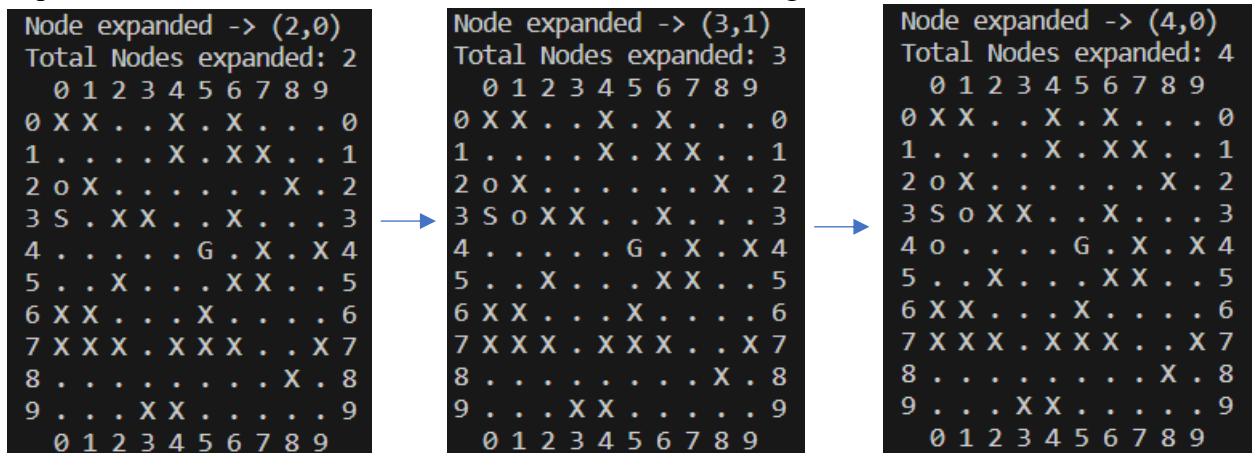
All the search algorithms use the dictionary to find the path from the starting point to the goal point if the goal state is expanded (meaning a path exists). All the search algorithms use a late goal test. Open cells are changed from '.' to 'o' when expanded.

Visualization

All maze prints show the column numbers (in blue underline) and row numbers (in orange) to help in locating a cell.

	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>
<u>0</u>	X	X	o	o	X	.	X	.	.	.
<u>1</u>	o	o	o	o	X	.	X	X	.	.
<u>2</u>	o	X	o	o	X	.
<u>3</u>	S	.	X	X	o	.	X	.	.	.
<u>4</u>	o		-	-	-	G	.	X	.	X
<u>5</u>	o	o	X	o	.	.	X	X	.	.
<u>6</u>	X	X	.	.	.	X
<u>7</u>	X	X	X	.	X	X	X	.	.	X
<u>8</u>	X	.
<u>9</u>	.	.	.	X	X

All search algorithms showcase each node expanded and the coordinates to aid in locating the expanded node. The below is from the breadth-first search algorithm.



If the goal state is expanded, the user is let known.

```

Node expanded -> (4,5)
Total Nodes expanded: 21
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S 0 X X 0 . X . . . 3
4 0 0 0 0 0 G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Solution found. Press enter to continue.

```

Then, a path from the starting point to the goal state is displayed.

```

Start state -> (3,0)
Solution Path Length Count: 0
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S 0 X X 0 . X . . . 3
4 0 0 0 0 0 G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Press enter to continue.

```



```

Node -> (3,1)
Solution Path Length Count: 1
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S - X X 0 . X . . . 3
4 0 0 0 0 0 G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Press enter to continue.

```



```

Node -> (4,1)
Solution Path Length Count: 2
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S - X X 0 . X . . . 3
4 0 | 0 0 0 G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Press enter to continue.

```



```

Node -> (4,4)
Solution Path Length Count: 5
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S - X X 0 . X . . . 3
4 0 | - - - G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Press enter to continue.

```



```

Node -> (4,3)
Solution Path Length Count: 4
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S - X X 0 . X . . . 3
4 0 | - - 0 G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Press enter to continue.

```



```

Node -> (4,2)
Solution Path Length Count: 3
 0 1 2 3 4 5 6 7 8 9
0 X X 0 0 X . X . . . 0
1 0 0 0 0 X . X X . . 1
2 0 X 0 0 . . . . X . 2
3 S - X X 0 . X . . . 3
4 0 | - 0 0 G . X . X 4
5 0 0 X 0 . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . . 9
 0 1 2 3 4 5 6 7 8 9
Press enter to continue.

```

Lastly, a summary of the search is displayed.

```
Goal reached -> (4,5)
Total Nodes Expanded: 21
Total Solution Path Length: 6
 0 1 2 3 4 5 6 7 8 9
0 X X o o X . X . . . 0
1 o o o o X . X X . . 1
2 o X o o . . . X . 2
3 S - X X o . X . . . 3
4 o | - - - G . X . X 4
5 o o X o . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . 9
 0 1 2 3 4 5 6 7 8 9
Solution: [(3, 0), (3, 1), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)]
Execution Time: 0.06643366813659668 seconds
Breadth first search completed.
```

A* search algorithm additionally displays an expanded node's estimate distance to the goal and number of steps from the starting point.

```
Node expanded -> (3,0)
Possible Solution Length: 6
Steps from start: 0
Total Nodes expanded: 1
 0 1 2 3 4 5 6 7 8 9
0 X X . . X . X . . . 0
1 . . . . X . X X . . 1
2 . X . . . . . X . 2
3 S . X X . . X . . . 3
4 . . . . . G . X . X 4
5 . . X . . . X X . . 5
6 X X . . . X . . . . 6
7 X X X . X X X . . X 7
8 . . . . . . . X . 8
9 . . . X X . . . . 9
 0 1 2 3 4 5 6 7 8 9
```

10x10 Maze Performance Table (10 Runs)

All mazes have 30% obstacle density.

Run	Algorithm	Solution Path Length	Nodes Expanded	Execution Time (in seconds)	Status
1	DFS	36	52	.16	Pass
2	DFS	0	15	.03	Fail
3	DFS	39	47	.25	Pass
4	DFS	40	42	.18	Pass
5	DFS	16	20	.08	Pass
6	DFS	20	33	.18	Pass
7	DFS	12	47	.18	Pass
8	DFS	31	46	.26	Pass
9	DFS	28	46	.18	Pass
10	DFS	24	27	.08	Pass
Average	DFS	24.6	37.5	.158	
1	BFS	6	21	.07	Pass
2	BFS	0	15	.00	Fail
3	BFS	9	37	.14	Pass
4	BFS	4	21	.02	Pass
5	BFS	10	50	.12	Pass
6	BFS	18	62	.26	Pass
7	BFS	12	62	.15	Pass
8	BFS	9	44	.11	Pass
9	BFS	10	64	.13	Pass
10	BFS	6	22	.08	Pass
Average	BFS	8.4	39.8	.108	Pass
1	A*	6	8	.03	Pass
2	A*	0	15	.06	Fail
3	A*	9	11	.10	Pass
4	A*	4	5	.03	Pass
5	A*	10	23	.13	Pass
6	A*	18	50	.18	Pass
7	A*	12	19	.03	Pass
8	A*	9	16	.10	Pass
9	A*	10	13	.05	Pass
10	A*	6	8	.01	Pass
Average	A*	8.4	16.8	.072	

Summary of Algorithm Performance (10 Runs)

Depth-First Search (DFS)

- Failures: **1**
- Successes: **9**
- Average Solution Path Length: **24.6**
- Average Nodes Expanded: **37.5**
- Average Execution Time: **0.158 seconds**

Breadth-First Search (BFS)

- Failures: **1**
- Successes: **9**
- Average Solution Path Length: **8.4**
- Average Nodes Expanded: **39.8**
- Average Execution Time: **0.108**

A* Algorithm

- Failures: **1**
- Successes: **9**
- Average Solution Path Length: **8.4**
- Average Nodes Expanded: **16.8**
- Average Execution Time: **0.072**

Observations

- Failure due to obstacle blockage between starting point and goal point.
- Breadth-first search's solution is optimal since path costs are uniform.
- Breadth-first search's count of nodes expanded is the highest since many runs had a great distance between starting point and goal point.
- Depth-first search's average execution time is highest since its average number of nodes expanded is like breadth-first search, but the solution path is triple the length.
- A* finds the optimal solution the quickest since it expands nodes "closest" to the goal.

Search End Result Visualization

Maze	DFS	BFS	A*
<pre> 0 1 2 3 4 5 6 7 8 9 0 XX . . X . X . . . 0 1 X . XX . . 1 2 . X X . 2 3 S . XX . . X . . . 3 4 G . X . X 4 5 . . X . . . XX . . 5 6 XX . . . X 6 7 XXX . XXX . . X 7 8 X . 8 9 . . . XX 9 0 1 2 3 4 5 6 7 8 9 </pre>	<p>Goal reached -> (4,5) Total Nodes Expanded: 52 Total Solution Path Length: 36 0 1 2 3 4 5 6 7 8 9 0 XX - X o X o o o 0 1 - - X o X X o o 1 2 X . - - - X o 2 3 S . XX - X - o 3 4 . . . - G . X X 4 5 . . X . . XX - 5 6 X X . . X o o - 6 7 X X X X X X - X 7 8 . . . - - - X o 8 9 . . . XX o - o o 9 0 1 2 3 4 5 6 7 8 9 Solution: [(3, 0), (2, 0), (1, 0), (1, 1), (, 8), (7, 8), (7, 7), (8, 7), (9, 7), (9, 6) Execution Time: 0.16059541702270508 seconds Depth first search completed.</p>	<p>Goal reached -> (4,5) Total Nodes Expanded: 21 Total Solution Path Length: 6 0 1 2 3 4 5 6 7 8 9 0 XX o o X . X . . . 0 1 0 0 0 0 X . X X . . 1 2 0 X o o X . 2 3 S - X X o . X . . . 3 4 0 - - - G . X . X 4 5 0 o X o . . X X . . 5 6 X X . . . X 6 7 X X X . X X X . . X 7 8 X . 8 9 . . . XX 9 0 1 2 3 4 5 6 7 8 9 Solution: [(3, 0), (3, 1), (4, 1), (4, 2), Execution Time: 0.06643366813659668 seconds Breadth first search completed.</p>	<p>Goal reached -> (4,5) Total Nodes Expanded: 8 Total Solution Path Length: 6 0 1 2 3 4 5 6 7 8 9 0 XX . . X . X . . . 0 1 X . XX . . 1 2 . X X . 2 3 S - X X . . X . . . 3 4 0 - - - G . X . X 4 5 . . X . . . XX . . 5 6 XX . . . X 6 7 XX X . XX X . . X 7 8 X . 8 9 . . . XX 9 0 1 2 3 4 5 6 7 8 9 Solution: [(3, 0), (3, 1), (4, 1), (4, 2), Execution Time: 0.03430604934692383 seconds A* search completed.*</p>
<pre> 0 1 2 3 4 5 6 7 8 9 0 . . . X . X . X . . 0 1 . . X XX . 1 2 X . . XX . X . . . 2 3 . . X . XX . S . X 3 4 . . . XX X 4 5 . . . XX XX . X . 5 6 X . . XX X . XX X 6 7 7 8 XX X . 8 9 X G 9 0 1 2 3 4 5 6 7 8 9 </pre>	<p>Node expanded -> (3,6) Total Nodes expanded: 15 0 1 2 3 4 5 6 7 8 9 0 . . . X . X . X o o 0 1 . . X XX o 1 2 X . . XX . X o o o 2 3 . . X . XX o S o X 3 4 . . . XX X o o o o 4 5 . . . XX X X o X o 5 6 X . . XX X . XX X 6 7 7 8 XX X . 8 9 X G 9 0 1 2 3 4 5 6 7 8 9 No path was found. Execution Time: 0.028409242630004883 seconds Depth first search completed.</p>	<p>Node expanded -> (0,8) Total Nodes expanded: 15 0 1 2 3 4 5 6 7 8 9 0 . . . X . X . X o o 0 1 . . X XX o 1 2 X . . XX . X o o o 2 3 . . X . XX o S o X 3 4 . . . XX X o o o o 4 5 . . . XX X X o X o 5 6 X . . XX X . XX X 6 7 7 8 XX X . 8 9 X G 9 0 1 2 3 4 5 6 7 8 9 No path was found. Execution Time: 0.0 seconds Breadth first search completed</p>	<p>Node expanded -> (0,8) Possible Solution Length: 16 Steps from start: 6 Total Nodes expanded: 15 0 1 2 3 4 5 6 7 8 9 0 . . . X . X . X o o 0 1 . . X XX o 1 2 X . . XX . X o o o 2 3 . . X . XX o S o X 3 4 . . . XX X o o o o 4 5 . . . XX X X o X o 5 6 X . . XX X . XX X 6 7 7 8 XX X . 8 9 X G 9 0 1 2 3 4 5 6 7 8 9 No path was found. Execution Time: 0.055417776107788086 seconds A* search completed.*</p>
<pre> 0 1 2 3 4 5 6 7 8 9 0 X . XX S 0 1 . . . X . X . . X 1 2 . XX . X . X . . . 2 3 . . X X 3 4 . . X . X 4 5 . . G . . X . . . 5 6 X . 6 7 X . XX . XXX . . 7 8 X X . 8 9 X . . . X 9 0 1 2 3 4 5 6 7 8 9 </pre>	<p>Goal reached -> (5,2) Total Nodes Expanded: 47 Total Solution Path Length: 39 0 1 2 3 4 5 6 7 8 9 0 X o X X S - - - - o 0 1 - - - X . X o X 1 2 XX X . X o o 2 3 - X . - - X 3 4 . X X . - 4 5 . G - X - 5 6 . . . - - X 6 7 X . XX . XX X - 7 8 X X o 8 9 X . . X o 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 4), (0, 5), (0, 6), (0, 7), (, 6), (4, 6), (5, 6), (6, 6), (6, 5), (6, 4) Execution Time: 0.24736261367797852 seconds Depth first search completed.</p>	<p>Goal reached -> (5,2) Total Nodes Expanded: 37 Total Solution Path Length: 9 0 1 2 3 4 5 6 7 8 9 0 X . XX S - o o o o 0 1 . . o o X X o o X 1 2 . XX o X X o o o 2 3 . . X - - o o o X 3 4 . . X X o o o o 4 5 . . G o X o o o 5 6 . . . o o o X . 6 7 X . XX . XX X . 7 8 X X . 8 9 X . . X . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 4), (0, 5), (1, 5), (2, 5), Execution Time: 0.1417379379272461 seconds Breadth first search completed.</p>	<p>Goal reached -> (5,2) Total Nodes Expanded: 11 Total Solution Path Length: 9 0 1 2 3 4 5 6 7 8 9 0 X . XX S 0 1 . . . X X . . X 1 2 . XX . X X . . 2 3 . . X - - . . X 3 4 . . X X o . . . 4 5 . . G . X . . . 5 6 X . 6 7 X . XX . XX X . 7 8 X X . 8 9 X . . X . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 4), (0, 5), (1, 5), (2, 5), Execution Time: 0.10047030448913574 seconds A* search completed.*</p>
<pre> 0 1 2 3 4 5 6 7 8 9 0 X . X . . 0 1 X . . . XX XX X 1 2 . . . X X 2 3 . S . . . G X . X 3 4 . . XX . . X . . X 4 5 X X 5 6 X . . XX X . X X 6 7 X 7 8 X X . 8 9 . XX XX X 9 0 1 2 3 4 5 6 7 8 9 </pre>	<p>Goal reached -> (3,5) Total Nodes Expanded: 42 Total Solution Path Length: 40 0 1 2 3 4 5 6 7 8 9 0 . - - X . X . . 0 1 X - X X X X . 1 2 . X - - - X . 2 3 . S - - G X X . 3 4 . . XX - X - X 4 5 X . - - . . X 5 6 X . . XX X . X X 6 7 . . - - X . X 7 8 X . - X - X o 8 9 . XX XX . - - X 9 0 1 2 3 4 5 6 7 8 9 Solution: [(3, 1), (2, 1), (1, 1), (0, 1), (, 8), (6, 8), (7, 8), (8, 8), (9, 8), (9, 7), (3, 5)] Execution Time: 0.17573785781860352 seconds Depth first search completed.</p>	<p>Goal reached -> (3,5) Total Nodes Expanded: 21 Total Solution Path Length: 4 0 1 2 3 4 5 6 7 8 9 0 0 0 0 . X . . X . 0 1 X o o o X X X X X 1 2 0 0 0 X o . . . X . 2 3 0 S - - - G X . X 3 4 0 o X X . . X . X 4 5 X o o X 5 6 X o . XX X . X . X 6 7 X . X 7 8 X X . 8 9 . XX XX X 9 0 1 2 3 4 5 6 7 8 9 Solution: [(3, 1), (3, 2), (3, 3), (3, 4), (Execution Time: 0.015376567840576172 seconds Breadth first search completed.</p>	<p>Goal reached -> (3,5) Total Nodes Expanded: 5 Total Solution Path Length: 4 0 1 2 3 4 5 6 7 8 9 0 X . X . . 0 1 X . . . XX XX X 1 2 . . . X X 2 3 . S - - - G X . X 3 4 . . XX . . X . . X 4 5 X X 5 6 X . . XX X . X X 6 7 X . X 7 8 X X . 8 9 . XX XX X 9 0 1 2 3 4 5 6 7 8 9 Solution: [(3, 1), (3, 2), (3, 3), (3, 4), (Execution Time: 0.029038190841674805 seconds A* search completed.*</p>

Latif Ahmed
Maze

<pre> 0 1 2 3 4 5 6 7 8 9 0 X . X X 0 1 . S X X X 1 2 . . . X . X X . X . 2 3 X X X 3 4 . . X X . 4 5 G . 5 6 . . . X 6 7 . . X 7 8 X . . X X 8 9 X X . . 9 0 1 2 3 4 5 6 7 8 9 </pre>	<pre> Goal reached -> (5,7) Total Nodes Expanded: 20 Total Solution Path Length: 16 0 1 2 3 4 5 6 7 8 9 0 X O X X - - - 0 0 0 1 . S X . . . X X 1 2 . - X X X X . 2 3 X . - - . X X 3 4 . . X . . . X . 4 5 G . . 5 6 . . . X 6 7 . . X 7 8 X . . X . . . X 8 9 X X . . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(1, 1), (2, 1), (2, 2), (3, 2), Execution Time: 0.07782101631164551 seconds Depth first search completed. </pre>	<pre> Goal reached -> (5,7) Total Nodes Expanded: 50 Total Solution Path Length: 10 0 1 2 3 4 5 6 7 8 9 0 X O X X O O O . . 0 1 O S X O O O O O X X 1 2 O - X O X X O X . 2 3 X O - - - - X X 3 4 O O X O O O X . 4 5 O O O O O O G . . 5 6 O O O X O 6 7 O O X . O 7 8 X O O X X 8 9 X O . . . X . . . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(1, 1), (2, 1), (2, 2), (3, 2), Execution Time: 0.12446427345275879 seconds Breadth first search completed. </pre>	<pre> Goal reached -> (5,7) Total Nodes Expanded: 23 Total Solution Path Length: 10 0 1 2 3 4 5 6 7 8 9 0 X . X X 0 1 . S X X X 1 2 . - X . X X . X . 2 3 X O - - - - X X 3 4 . O X O O O X . 4 5 . O O O O O G . . 5 6 . . . X 6 7 . . X 7 8 X . . X X 8 9 X X . . . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(1, 1), (2, 1), (2, 2), (3, 2), Execution Time: 0.12897825241088867 seconds A* search completed.* </pre>
<pre> 0 1 2 3 4 5 6 7 8 9 0 X X X . 0 1 . X X X X S . X X . 1 2 . . X . X . X . . 2 3 X . X . 3 4 X X . X X . 4 5 . X . X . X X X . . 5 6 X . X . . 6 7 . . . X X X . X . 7 8 X X . . X 8 9 . . X . . X . G . 9 0 1 2 3 4 5 6 7 8 9 </pre>	<pre> Goal reached -> (9,7) Total Nodes Expanded: 33 Total Solution Path Length: 20 0 1 2 3 4 5 6 7 8 9 0 X X O O O O O O X O 0 1 . X X X X S O X X O 1 2 . . X . X X - - 2 3 X X 3 4 X X . X . - X 4 5 . X . X . X X X O 5 6 . . . X . X . - 6 7 . . X X X . X O 7 8 X X - X 8 9 . . X . . X G O 9 0 1 2 3 4 5 6 7 8 9 Solution: [(1, 5), (2, 5), (3, 5), (4, 5), (8, 9), (7)] Execution Time: 0.17658066749572754 seconds Depth first search completed. </pre>	<pre> Goal reached -> (9,7) Total Nodes Expanded: 62 Total Solution Path Length: 18 0 1 2 3 4 5 6 7 8 9 0 X X O O O O O O X O 0 1 O X X X X S O X X O 1 2 O O X O X X - - 2 3 O O O O O X X 3 4 X X O X O - X 4 5 O X O X O X X X O 5 6 O O O O X . X . - 6 7 O O O X X X O X O 7 8 X O O O O X O X 8 9 O O X O O X . G . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(1, 5), (2, 5), (3, 5), (4, 5), Execution Time: 0.2629420757293701 seconds Breadth first search completed. </pre>	<pre> Goal reached -> (9,7) Total Nodes Expanded: 50 Total Solution Path Length: 18 0 1 2 3 4 5 6 7 8 9 0 X X O O O O O O X . 0 1 . X X X X S O X X . 1 2 . . X O X X - - 2 3 . . O O O X X 3 4 X X O X O - X 4 5 . X O X O X X X . 5 6 . . O O X . X . O 6 7 . . O O X X X . X O 7 8 X O O O O X X 8 9 . O X O O X . G . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(1, 5), (2, 5), (3, 5), (4, 5), Execution Time: 0.17923617362976074 seconds A* search completed.* </pre>
<pre> 0 1 2 3 4 5 6 7 8 9 0 X X . X X G X . X . 0 1 . . . X 1 2 . . . X X 2 3 X X . X X 3 4 . . X X . 4 5 X X . . . 5 6 X . X . . X X X . 6 7 X . X 7 8 . X . X . . . X X 8 9 X . X X S . . X X . 9 0 1 2 3 4 5 6 7 8 9 </pre>	<pre> Goal reached -> (0,5) Total Nodes Expanded: 47 Total Solution Path Length: 12 0 1 2 3 4 5 6 7 8 9 0 X X O X X G X O X O 0 1 O O O O X - O O O 1 2 O O O X X . O O O 2 3 X O O O O X X X X 3 4 O O X O - - X . 4 5 X O O O X . . . 5 6 X O X O X . X X . 6 7 O O O O . . . X 7 8 O X O X . . X . X 8 9 X . X X S . . X X . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 4), (8, 4), (7, 4), (6, 4), Execution Time: 0.17689251899719238 seconds Depth first search completed. </pre>	<pre> Goal reached -> (0,5) Total Nodes Expanded: 62 Total Solution Path Length: 12 0 1 2 3 4 5 6 7 8 9 0 X X O X X G X O X . 0 1 O O O O X - O O . 1 2 O O O X O O O . 2 3 X O O O O X X X X 3 4 O O X O - - O X 4 5 X O O O X O O O 5 6 X O X O X O X X 6 7 O O O O O O O X 7 8 O X O X O O X X 8 9 X . X X S O O X X . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(9, 4), (8, 4), (7, 4), (6, 4), Execution Time: 0.14533138275146484 seconds Breadth first search completed. </pre>	<pre> Goal reached -> (0,5) Total Nodes Expanded: 19 Total Solution Path Length: 12 0 1 2 3 4 5 6 7 8 9 0 X X . X X G X . X . 0 1 . . . X 1 2 . . . X X 2 3 X . . O X X X X 3 4 . . X O - . X . 4 5 X . . . X . . . 5 6 X . X . X . X X . 6 7 O . . X 7 8 . X . X O . X X 8 9 X . X X S O . X X . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(9, 4), (8, 4), (7, 4), (6, 4), Execution Time: 0.031032085418701172 seconds A* search completed.* </pre>
<pre> 0 1 2 3 4 5 6 7 8 9 0 . . . X . . S . . 0 1 X . . X 1 2 X X X . X 2 3 X . X . 3 4 X . . . X . . . X . 4 5 . X 5 6 . X . X . . X X X 6 7 . . . X G . . X . 7 8 . X 8 9 X . X X . . X . . 9 0 1 2 3 4 5 6 7 8 9 </pre>	<pre> Goal reached -> (7,4) Total Nodes Expanded: 46 Total Solution Path Length: 31 0 1 2 3 4 5 6 7 8 9 0 . . . X O O S - - 0 1 X . . X O O - O 1 2 X X X . X - O 2 3 X X 3 4 X . . . X - X 4 5 . X . . . - . 5 6 . X . X . X X X X 6 7 . . . X G - X O 7 8 . X . . - O O 8 9 X . X X - X O O 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 6), (0, 7), (0, 8), (0, 9), (5), (4, 6), (5, 6), (5, 5), (6, 5), (7, 5) Execution Time: 0.26088428497314453 seconds Depth first search completed. </pre>	<pre> Goal reached -> (7,4) Total Nodes Expanded: 44 Total Solution Path Length: 9 0 1 2 3 4 5 6 7 8 9 0 . . . X O O S O O O 0 1 X . . X O O O O O 1 2 X X X O X - O O O 2 3 . . O O O X O X O 3 4 X . O O X O O X O 4 5 . X . O O O O O 5 6 . X . X O X X X X 6 7 . . . X G O X . . 7 8 . X 8 9 X . X X . . X . . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 6), (1, 6), (2, 6), (2, 5), Execution Time: 0.11174225807189941 seconds Breadth first search completed. </pre>	<pre> Goal reached -> (7,4) Total Nodes Expanded: 16 Total Solution Path Length: 9 0 1 2 3 4 5 6 7 8 9 0 . . . X O - S . . . 0 1 X . . X O O . . . 1 2 X X X . X O . . . 2 3 X . X . 3 4 X . . . X . X . 4 5 . X 5 6 . X . X O X X X 6 7 . . . X G . . X . 7 8 . X 8 9 X . X X . . X . . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(0, 6), (0, 5), (1, 5), (2, 5), Execution Time: 0.10463619232177734 seconds A* search completed.* </pre>
<pre> 0 1 2 3 4 5 6 7 8 9 0 . . G . . X . . . 0 1 . . X . . X X . X X 1 2 X 2 3 . X . . X . X X . 3 4 . X . . . X . . . 4 5 . X S . X 5 6 X . X . X X X . X 6 7 X X . . . 7 8 X . . . X 8 9 X X . . 9 0 1 2 3 4 5 6 7 8 9 </pre>	<pre> Goal reached -> (0,2) Total Nodes Expanded: 46 Total Solution Path Length: 28 0 1 2 3 4 5 6 7 8 9 0 . . G O X O O O O 0 1 . . X O X X O X X 1 2 . . . X O O O O O 2 3 . X . X O X X O O 3 4 . X . . X . - 4 5 . X . . . S X 5 6 X . X X X X O X 6 7 X . . X - - 7 8 X . . X - - 8 9 . . . - X X - 9 0 1 2 3 4 5 6 7 8 9 Solution: [(5, 7), (4, 7), (4, 8), (5, 8), (3), (7, 3), (6, 3), (5, 3), (4, 3), (3, 3) Execution Time: 0.18412542343139648 seconds Depth first search completed. </pre>	<pre> Goal reached -> (0,2) Total Nodes Expanded: 64 Total Solution Path Length: 10 0 1 2 3 4 5 6 7 8 9 0 . . G O X O O O O 0 1 . . X O X X O X X 1 2 . . O X O O O O O 2 3 . X O X O X X O O 3 4 . X O - X O O O 4 5 . X O O - - S O X 5 6 X O X O X X X O X 6 7 X O O O O X O O 7 8 X O O O X O O O 8 9 . O O O O X X O O 9 0 1 2 3 4 5 6 7 8 9 Solution: [(5, 7), (5, 6), (5, 5), (5, 4), Execution Time: 0.13158082962036133 seconds Breadth first search completed. </pre>	<pre> Goal reached -> (0,2) Total Nodes Expanded: 13 Total Solution Path Length: 10 0 1 2 3 4 5 6 7 8 9 0 . . G . X . . . 0 1 . . X . X X . X X 1 2 . . . X 2 3 . X . X . X X . 3 4 . X . - X O . . 4 5 . X . - - S . X 5 6 X . X . X X X . X 6 7 X . . . X 7 8 X . . . X 8 9 X X . . 9 0 1 2 3 4 5 6 7 8 9 Solution: [(5, 7), (5, 6), (5, 5), (5, 4), Execution Time: 0.04979586601257324 seconds A* search completed.* </pre>

<pre> 0 1 2 3 4 5 6 7 8 9 0 X X X X 0 1 X X 1 2 X X . . X X X 2 3 G X X 3 4 X X . X X . 4 5 X X S 5 6 X . . X X . X . . . 6 7 . X . X 7 8 X . X . . . X X . . 8 9 . . . X X X 9 0 1 2 3 4 5 6 7 8 9 </pre>	<pre> Goal reached -> (3,7) Total Nodes Expanded: 27 Total Solution Path Length: 24 0 1 2 3 4 5 6 7 8 9 0 X X - X X 0 1 X . o X 1 2 X X . . X . X X 2 3 G X X 3 4 X X X X o 4 5 X X - S 5 6 X . . X X X 6 7 . X . X . - - 7 8 X . X . . X X - 8 9 . . . X . . . X X 9 0 1 2 3 4 5 6 7 8 9 Solution: [(5, 9), (6, 9), (7, 9), (8, 9), (9, 9), (9, 8), (9, 7), (9, 6), (8, 6), (8, 5), (8, 4), (8, 3), (8, 2), (8, 1), (8, 0), (7, 0), (7, 1), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6), (7, 7), (7, 8), (7, 9), (6, 9), (6, 8), (6, 7), (6, 6), (6, 5), (6, 4), (6, 3), (6, 2), (6, 1), (6, 0), (5, 0), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7), (5, 8), (5, 9), (4, 9), (4, 8), (4, 7), (4, 6), (4, 5), (4, 4), (4, 3), (4, 2), (4, 1), (4, 0), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8), (3, 9), (2, 9), (2, 8), (2, 7), (2, 6), (2, 5), (2, 4), (2, 3), (2, 2), (2, 1), (2, 0), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9), (0, 9), (0, 8), (0, 7), (0, 6), (0, 5), (0, 4), (0, 3), (0, 2), (0, 1), (0, 0)] Execution Time: 0.08276772499084473 seconds Depth first search completed. </pre>	<pre> Goal reached -> (3,7) Total Nodes Expanded: 22 Total Solution Path Length: 6 0 1 2 3 4 5 6 7 8 9 0 X X X X 0 1 X X 1 2 X X . . X X X 2 3 G X X 3 4 X X X X o 4 5 X X - S 5 6 X . . X X X 6 7 . X . X 7 8 X . X . . X X o o 8 9 . . . X . . . X X 9 0 1 2 3 4 5 6 7 8 9 Solution: [(5, 9), (5, 8), (5, 7), (5, 6), (5, 5), (5, 4), (5, 3), (5, 2), (5, 1), (5, 0), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (3, 9), (3, 8), (3, 7), (3, 6), (3, 5), (3, 4), (3, 3), (3, 2), (3, 1), (3, 0), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9), (1, 9), (1, 8), (1, 7), (1, 6), (1, 5), (1, 4), (1, 3), (1, 2), (1, 1), (1, 0), (0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (0, 7), (0, 8), (0, 9)] Execution Time: 0.08047795295715332 seconds Breadth first search completed. </pre>	<pre> Goal reached -> (3,7) Total Nodes Expanded: 8 Total Solution Path Length: 6 0 1 2 3 4 5 6 7 8 9 0 X X X X 0 1 X X 1 2 X X . . X X X 2 3 G X X 3 4 X X X X o 4 5 X X - S 5 6 X . . X X . X . . . 6 7 . X . X 7 8 X . X . . X X . . 8 9 . . . X . . . X X 9 0 1 2 3 4 5 6 7 8 9 Solution: [(5, 9), (5, 8), (5, 7), (5, 6), (5, 5), (5, 4), (5, 3), (5, 2), (5, 1), (5, 0), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (3, 9), (3, 8), (3, 7), (3, 6), (3, 5), (3, 4), (3, 3), (3, 2), (3, 1), (3, 0), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9), (1, 9), (1, 8), (1, 7), (1, 6), (1, 5), (1, 4), (1, 3), (1, 2), (1, 1), (1, 0), (0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (0, 7), (0, 8), (0, 9)] Execution Time: 0.007682323455810547 seconds A* search completed.* </pre>
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